Eur. Phys. J. C 4, 717–721 (1998)
DOI 10.1007/s100529800792

THE EUROPEAN PHYSICAL JOURNAL C

The vector-tensor multiplet in harmonic superspace

Norbert Dragon, Sergei M. Kuzenko*, Ulrich Theis

Institut für Theoretische Physik, Universität Hannover, Appelstraße 2, D-30167 Hannover, Germany

Received: 10 October 1997 / Published online: 23 February 1998

Abstract. We describe the vector-tensor multiplet and derive its Chern-Simons coupling to the $N=2$ Yang-Mills gauge superfield in harmonic superspace.

The $N=2$ vector-tensor multiplet, which was discovered many years ago by Sohnius, Stelle and West [1] and then forgotten for a while, has recently received much interest [2–4] due to the fact that it originates in the low-energy effective Lagrangian of $N=2$ heterotic string vacua. As a representation of $N=2$ supersymmetry, this multiplet is very similar to the massless $8+8$ Fayet-Sohnius hypermultiplet [5,6] which possesses an off-shell central charge generating the equations of motion.

The vector-tensor multiplet is the only known $N=2$, $D=4$ supersymmetric model that has not yet been formulated in the harmonic superspace [7]. Since the harmonic superspace is believed to be a universal framework for $N=2$ supersymmetric theories, finding a relevant formulation for the vector-tensor multiplet seems to be of principal importance. On the other hand, adequate formulations of the vector-tensor multiplet in an $N=2$ superspace with central charges have been given in recent papers [8,9]. Our primary goal in this letter is to show that the main results of [8,9] have a natural origin in the harmonic superspace approach.

We start with re-formulating the Sohnius prescription of constructing supersymmetric actions [6] in harmonic superspace. The harmonic central charge superspace [7] extends the $N=2$ central charge superspace [6], with coordinates $\{x^m, z, \theta^\alpha_i, \bar\theta^i_{\dot\alpha}\}$, $\overline{\theta^\alpha_i} = \bar\theta^{\dot\alpha\, i}$ (where z is the central charge real variable), by the two-sphere $S^2 = SU(2)/U(1)$ parameterized by harmonics, i.e. group elements

$$
\begin{gathered}
({u_i}^-\,,\ {u_i}^+) \in SU(2) \\
u^+_i = \varepsilon_{ij} u^{+j} \qquad \overline{u^{+i}} = u^-_i \qquad u^{+i}u^-_i = 1\;.
\end{gathered}
\tag{1}
$$

The analytic basis of the harmonic superspace defined by

$$
\begin{aligned}
x^m_A &= x^m - 2\mathrm{i}\theta^{(i}\sigma^m\bar\theta^{j)}u^+_i u^-_j \\
z_A &= z + \mathrm{i}(\theta^{+\alpha}\theta^-_\alpha - \bar\theta^+_{\dot\alpha}\bar\theta^{-\dot\alpha}) \\
\theta^\pm_\alpha &= u^\pm_i \theta^i_\alpha \\
\bar\theta^\pm_{\dot\alpha} &= u^\pm_i \bar\theta^i_{\dot\alpha}
\end{aligned}
\tag{2}
$$

is most suitable to the description of analytic superfields $\Phi(\zeta, u)$ which depends only on the variables

$$
\zeta^M \equiv \{x^m_A, z_A, \theta^{+\alpha}, \bar\theta^+_{\dot\alpha}\}
\tag{3}
$$

and harmonics $u^\pm_i$ (the original basis of the harmonic superspace is called central [7]). Below we will mainly work in the analytic basis and omit the corresponding subscript "A". The explicit expressions for the covariant derivatives $D^\pm_\alpha = D^i_\alpha u^\pm_i$, $\bar D^\pm_{\dot\alpha} = \bar D^i_{\dot\alpha} u^\pm_i$ in the analytic basis can be found in [7].

The GIKOS rule [7] of constructing $N=2$ supersymmetric actions

$$
\int d\zeta^{(-4)} du\, \mathcal{L}^{(4)} \qquad\qquad d\zeta^{(-4)} = d^4x d^2\theta^+ d^2\bar\theta^+
\tag{4}
$$

involves an analytic superfield $\mathcal{L}^{(4)}(\zeta, u)$ of $U(1)$-charge $+4$ which is invariant (up to derivatives) under central charge transformations generated by $\partial_z \equiv \partial/\partial z$

$$
\frac{\partial}{\partial z}\mathcal{L}^{(4)} = \frac{\partial}{\partial x^m} f^{(4)m}\;.
\tag{5}
$$

Here $\mathcal{L}^{(4)}$ is a function of the dynamical superfields, their covariant derivatives and, in general, of the harmonic variables.

In harmonic superspace there exists a prescription to construct invariant actions even for non-vanishing central charges. The construction makes use of a constrained analytic superfield $\mathcal{L}^{++}(\zeta, u)$. $\mathcal{L}^{++}$ is an analytic superfield of $U(1)$-charge $+2$

$$
D^+_\alpha \mathcal{L}^{++} = \bar D^+_{\dot\alpha}\mathcal{L}^{++} = 0
\tag{6}
$$

which satisfies the covariant constraint

$$
D^{++}_{\mathrm{c}} \mathcal{L}^{++} = 0\;.
\tag{7}
$$

* Alexander von Humboldt Research Fellow. On leave from Department of Quantum Field Theory, Tomsk State University, Tomsk 634050, Russia

$D_{\rm c}^{++}$ acts according to

$$D_{\rm c}^{++} = D^{++} + {\rm i}\,(\theta^{+\alpha}\theta^{+}_{\alpha} - \bar\theta^{+}_{\dot\alpha}\bar\theta^{+\dot\alpha})\frac{\partial}{\partial z}$$
$$D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} - 2{\rm i}\,\theta^{+}\sigma^{m}\bar\theta^{+}\frac{\partial}{\partial x^{m}} \tag{8}$$

on analytic superfields. Then the action

$$S = \int d\zeta^{(-4)}du\,\left((\theta^{+})^{2} - (\bar\theta^{+})^{2}\right)\mathcal{L}^{++} \tag{9}$$

is supersymmetric and, hence, invariant under central charge transformations. Under a supersymmetry transformation

$$\delta x^{m} = -2{\rm i}\,(\epsilon^{-}\sigma^{m}\bar\theta^{+} + \theta^{+}\sigma^{m}\bar\epsilon^{-})$$
$$\delta z = 2{\rm i}\,(\epsilon^{-}\bar\theta^{+} - \bar\epsilon^{-}\bar\theta^{+})$$
$$\delta\theta^{+}_{\alpha} = \epsilon^{+}_{\alpha} \qquad \delta\bar\theta^{+}_{\dot\alpha} = \bar\epsilon^{+}_{\dot\alpha} \tag{10}$$

S changes by

$$\delta S = \int d\zeta^{(-4)}du\left\{\left((\theta^{+})^{2} - (\bar\theta^{+})^{2}\right)\delta z\frac{\partial}{\partial z}\mathcal{L}^{++}\right.$$
$$\left. -2\left(\epsilon^{+}\theta^{+} - \bar\epsilon^{+}\bar\theta^{+}\right)\mathcal{L}^{++}\right\}. \tag{11}$$

Making use of the identity $2\left(\epsilon^{+}\theta^{+} - \bar\epsilon^{+}\bar\theta^{+}\right) = -{\rm i}\,D^{++}\delta z$ and integrating by parts in (11), one arrives at

$$\delta S = -{\rm i}\int d\zeta^{(-4)}du\,\delta z D_{\rm c}^{++}\mathcal{L}^{++} \tag{12}$$

and this is equal to zero due to (7). The action (9) is real if $\mathcal{L}^{++}$ is imaginary

$$\breve{\mathcal{L}}^{++} = -\mathcal{L}^{++} \tag{13}$$

with respect to the analyticity preserving conjugation (smile) $\breve{\ } = \bar{\ }^{\star}$ introduced in [7], where the operation $\bar{\ }$ denotes the complex conjugation and the operation * is defined by $(u^{+}_{i})^{\star} = u^{-}_{i}$, $(u^{-}_{i})^{\star} = -u^{+}_{i}$, hence $(u^{\pm}_{i})^{\star\star} = -u^{\pm}_{i}$.

Equation (9) is the formulation of the Sohnius action [6] (see also [10]) in the harmonic superspace. More explicitly, in the central basis the constraint (7) means

$$\mathcal{L}^{++} = \mathcal{L}^{ij}(x,z,\theta)u^{+}_{i}u^{+}_{j} \tag{14}$$

for some u-independent superfields $\mathcal{L}^{ij}$, and the analyticity conditions (6) take the form

$$D^{(i}_{\alpha}\mathcal{L}^{jk)} = \bar D^{(i}_{\dot\alpha}\mathcal{L}^{jk)} = 0\,. \tag{15}$$

Since

$$d\zeta^{(-4)} = \frac{1}{16}d^{4}xD^{-\alpha}D^{-}_{\alpha}\bar D^{-}_{\dot\alpha}\bar D^{-\dot\alpha} \tag{16}$$

the action (9) turns, upon integrating over S^{2}, into

$$S = \frac{1}{12}\int d^{4}x\,\left(D^{\alpha i}D^{j}_{\alpha} - \bar D^{i}_{\dot\alpha}\bar D^{\dot\alpha j}\right)\mathcal{L}_{ij}\,. \tag{17}$$

It is instructive to consider two examples. The 8 + 8 Fayet-Sohnius off-shell hypermultiplet coupled to the $N = 2$ gauge multiplet is described in the $N = 2$ central charge superspace by a superfield $q_{i}(x,z,\theta)$ satisfying the constraints [6]

$$\mathcal{D}^{(i}_{\alpha}q^{j)} = \bar{\mathcal{D}}^{(i}_{\dot\alpha}q^{j)} = 0 \tag{18}$$

with $\mathcal{D}^{i}_{\alpha}$ the gauge covariant derivatives. This is equivalent to the fact that the superfield $q^{+} = q_{i}u^{+i}$ is covariantly analytic

$$\mathcal{D}^{+}_{\alpha}q^{+} = \bar{\mathcal{D}}^{+}_{\dot\alpha}q^{+} = 0 \tag{19}$$

and satisfies the gauge covariant constraint

$$\mathcal{D}_{\rm c}^{++}q^{+} = 0\,. \tag{20}$$

In the analytic basis $\mathcal{D}_{\rm c}^{++}$ is

$$\mathcal{D}_{\rm c}^{++} = \mathcal{D}^{++} + {\rm i}\,(\theta^{+\alpha}\theta^{+}_{\alpha} - \bar\theta^{+}_{\dot\alpha}\bar\theta^{+\dot\alpha})\frac{\partial}{\partial z}$$
$$\mathcal{D}^{++} = D^{++} + {\rm i}\,V^{++} \tag{21}$$

where V^{++} is the analytic Yang-Mills gauge prepotential [7]. Therefore, the gauge invariant superfield

$$\mathcal{L}^{++}_{\rm FS} = \frac{\rm i}{2}\left(\breve q^{+}\partial_{z}q^{+} - \partial_{z}\breve q^{+}q^{+}\right) + m\,\breve q^{+}q^{+} \tag{22}$$

meets the requirements (6) and (7). Because of (20), the corresponding action can be rewritten in the following form

$$S_{\rm FS} = \int d\zeta^{(-4)}du\,\left\{-\breve q^{+}\mathcal{D}^{++}q^{+}\right.$$
$$\left.+m\,\breve q^{+}q^{+}\left((\theta^{+})^{2} - (\bar\theta^{+})^{2}\right)\right\} \tag{23}$$

which is very similar to the action functional of the infinite-component q-hypermultiplet [7]. Another non-trivial example is the effective action of the $N = 2$ super Yang-Mills theory [11,12] (supersymmetry without central charges)

$$S_{\rm SYM} = {\rm tr}\int d^{4}xd^{4}\theta\mathcal{F}(W) + {\rm tr}\int d^{4}xd^{4}\bar\theta\bar{\mathcal{F}}(\bar W) \tag{24}$$

where W is the covariantly chiral field strength of the $N = 2$ gauge superfield [13]. $S_{\rm SYM}$ can be represented as follows

$$S_{\rm SYM} = \frac{1}{4}\,{\rm tr}\int d\zeta^{(-4)}du\,\left((\theta^{+})^{2} - (\bar\theta^{+})^{2}\right)\mathcal{L}^{++}_{\rm SYM}$$
$$\mathcal{L}^{++}_{\rm SYM} = \left(\mathcal{D}^{+}\right)^{2}\mathcal{F}(W) - \left(\bar{\mathcal{D}}^{+}\right)^{2}\bar{\mathcal{F}}(\bar W)\,. \tag{25}$$

It is obvious that $\mathcal{L}^{++}_{\rm SYM}$ satisfies the requirements (6) and (7).

A free vector-tensor multiplet can be described in the harmonic superspace by an analytic spinor superfield $\Psi^{+}_{\alpha}(\zeta,u)$

$$D^{+}_{\alpha}\Psi^{+}_{\beta} = \bar D^{+}_{\dot\alpha}\Psi^{+}_{\beta} = 0 \tag{26}$$

subject to the constraints

$$D_{\mathrm{c}}^{++}\Psi_\alpha^+ = 0 \tag{27}$$

$$D^{-\alpha}\Psi_\alpha^+ = \bar{D}^{-\dot\alpha}\breve{\Psi}_{\dot\alpha}^+ \tag{28}$$

with $\breve{\Psi}_{\dot\alpha}^+$ the smile-conjugate of Ψ_α^+. Equation (27) implies that in the central basis Ψ_α^+ reads

$$\Psi_\alpha^+ = \Psi_{\alpha i}(x,z,\theta)u^{+i} \qquad \breve{\Psi}_{\dot\alpha}^+ = -\bar{\Psi}_{\dot\alpha}^i(x,z,\theta)u_i^+ \tag{29}$$

for some u-independent superfields $\Psi_{\alpha i}$ and its complex conjugate $\bar{\Psi}_{\dot\alpha}^i$. Then, the analyticity conditions (26) are equivalent to

$$D_\alpha^{(i}\Psi_\beta^{j)} = \bar{D}_{\dot\alpha}^{(i}\Psi_\beta^{j)} = 0 \tag{30}$$

and the reality condition (28) takes the form

$$D^{\alpha i}\Psi_{\alpha i} = \bar{D}_{\dot\alpha i}\bar{\Psi}^{\dot\alpha i}\,. \tag{31}$$

Equations (30) and (31) constitute the constraints defining the field strengths of the free vector-tensor multiplet [8].

Using the anticommutation relations

$$\{D_\alpha^+, D_\beta^-\} = 2\mathrm{i}\,\varepsilon_{\alpha\beta}\partial_z \qquad \{\bar{D}_{\dot\alpha}^+, \bar{D}_{\dot\beta}^-\} = 2\mathrm{i}\,\varepsilon_{\dot\alpha\dot\beta}\partial_z$$
$$\{D_\alpha^+, \bar{D}_{\dot\beta}^-\} = -\{D_\alpha^-, \bar{D}_{\dot\beta}^+\} = -2\mathrm{i}\,\partial_{\alpha\dot\beta} \tag{32}$$

one immediately deduces from (26) and (28) generalized Dirac equations

$$\partial_z\Psi_\alpha^+ = -\partial_{\alpha\dot\beta}\breve{\Psi}^{+\dot\beta} \qquad \partial_z\breve{\Psi}_{\dot\alpha}^+ = \partial_{\beta\dot\alpha}\Psi^{+\beta} \tag{33}$$

and hence

$$\partial_z^2\Psi_\alpha^+ = \Box\Psi_\alpha^+\,. \tag{34}$$

The last relation can be also obtained from (26) and (27), in complete analogy to the Fayet-Sohnius hypermultiplet. We read (33) and (34) as a definition of the central charge. If one had not allowed for a central charge then the constraints (26)–(28) would have restricted the multiplet to be on-shell.

The super Lagrangian associated with the vector-tensor multiplet reads

$$\mathcal{L}^{++}_{\mathrm{vt,free}} = -\frac{1}{4}\left(\Psi^{+\alpha}\Psi_\alpha^+ - \breve{\Psi}_{\dot\alpha}^+\breve{\Psi}^{+\dot\alpha}\right)\,. \tag{35}$$

Under central charge transformations it changes by derivatives

$$\partial_z\mathcal{L}^{++}_{\mathrm{vt,free}} = -\frac{1}{2}\partial_{\alpha\dot\alpha}\left(\Psi^{+\alpha}\breve{\Psi}^{+\dot\alpha}\right)\,. \tag{36}$$

The functional

$$S_{\mathrm{vt,free}} = \int d\zeta^{(-4)}du\,\left((\theta^+)^2 - (\bar\theta^+)^2\right)\mathcal{L}^{++}_{\mathrm{vt,free}} \tag{37}$$

can be seen to coincide with the action given in [8]. Another possible structure

$$\mathcal{L}^{++}_{\mathrm{der,free}} = -\frac{\mathrm{i}}{4}\left(\Psi^{+\alpha}\Psi_\alpha^+ + \breve{\Psi}_{\dot\alpha}^+\breve{\Psi}^{+\dot\alpha}\right) \tag{38}$$

produces a total derivative when integrated over the superspace.

The constraints (26)–(28) can be partially solved in terms of a real u-independent potential $L(x,z,\theta)$

$$\Psi_\alpha^+ = \mathrm{i}\,D_\alpha^+L \qquad \bar{L} = L \tag{39}$$

which is still restricted by

$$D_\alpha^+D_\beta^+L = D_\alpha^+\bar{D}_{\dot\beta}^+L = 0\,. \tag{40}$$

If one includes coupling to the $N=2$ Yang-Mills gauge superfield, described by the covariantly chiral strength W and its conjugate $\bar{W}$ [13], the constraints (40) can be consistently deformed as follows [9]

$$\mathcal{D}^{+\alpha}\mathcal{D}_\alpha^+\mathbb{L} = \kappa\,\mathrm{tr}\,\left(\bar{\mathcal{D}}_{\dot\alpha}^+\bar{W}\cdot\bar{\mathcal{D}}^{+\dot\alpha}\bar{W}\right) \tag{41}$$

$$\mathcal{D}_\alpha^+\bar{\mathcal{D}}_{\dot\beta}^+\mathbb{L} = -\kappa\,\mathrm{tr}\,\left(\mathcal{D}_\alpha^+W\cdot\bar{\mathcal{D}}_{\dot\beta}^+\bar{W}\right) \tag{42}$$

where $\mathbb{L}$ is a real u-independent gauge invariant superfield while W is invariant under the central charge. Such a deformation corresponds in particular to the Chern-Simons coupling of the antisymmetric tensor field, contained in the vector-tensor multiplet, to the Yang-Mills gauge field.

The independent components of the vector-tensor multiplet can be chosen as

$$\begin{gathered}
\Phi = \mathbb{L}\,| \qquad D = \partial_z\mathbb{L}\,| \\
\psi_\alpha^i = \mathcal{D}_\alpha^i\mathbb{L}\,| \qquad \bar\psi_{\dot\alpha i} = \bar{\mathcal{D}}_{\dot\alpha i}\mathbb{L}\,| \\
G_{\alpha\beta} = \tfrac{1}{2}[\mathcal{D}_{\alpha i}\,,\mathcal{D}_\beta^i]\,\mathbb{L}\,| \qquad \bar{G}_{\dot\alpha\dot\beta} = -\tfrac{1}{2}[\bar{\mathcal{D}}_{\dot\alpha i}\,,\bar{\mathcal{D}}_{\dot\beta}^i]\,\mathbb{L}\,| \\
H_{\alpha\dot\alpha} = \bar{H}_{\alpha\dot\alpha} = -\tfrac{1}{2}[\mathcal{D}_\alpha^i\,,\,\bar{\mathcal{D}}_{\dot\alpha i}]\,\mathbb{L}\,|
\end{gathered} \tag{43}$$

while the components of the vector multiplet are

$$\begin{gathered}
X = W\,| \qquad \bar{X} = \bar{W}\,| \\
\lambda_\alpha^i = \mathcal{D}_\alpha^iW\,| \qquad \bar\lambda_{\dot\alpha i} = \bar{\mathcal{D}}_{\dot\alpha i}\bar{W}\,| \\
F_{\alpha\beta} = -\tfrac{1}{4}[\mathcal{D}_{\alpha i}\,,\mathcal{D}_\beta^i]W\,| \qquad \bar{F}_{\dot\alpha\dot\beta} = \tfrac{1}{4}[\bar{\mathcal{D}}_{\dot\alpha i}\,,\bar{\mathcal{D}}_{\dot\beta}^i]\bar{W}\,| \\
Y^{ij} = -\tfrac{\mathrm{i}}{4}\left(\mathcal{D}^{\alpha i}\mathcal{D}_\alpha^jW + \bar{\mathcal{D}}_{\dot\alpha}^i\bar{\mathcal{D}}^{\dot\alpha j}\bar{W}\right)\,|
\end{gathered} \tag{44}$$

with F_{mn} the field strength associated with the Yang-Mills gauge field A_m. The fields H_m and G_{mn} are subject to the constraints

$$\begin{aligned}
\partial_mH^m &= \kappa\,\mathrm{tr}\left\{F^{mn}\tilde{F}_{mn} - \frac{1}{2}\partial_m(\lambda^i\sigma^m\bar\lambda_i)\right\} \\
\partial_m\tilde{G}^{mn} &= 2\kappa\,\partial_m\,\mathrm{tr}\left\{(X+\bar{X})\tilde{F}^{mn} + \frac{\mathrm{i}}{4}\lambda^i\sigma^{mn}\lambda_i\right. \\
&\qquad \left. + \frac{\mathrm{i}}{4}\bar\lambda_i\bar\sigma^{mn}\bar\lambda^i\right\}
\end{aligned} \tag{45}$$

which can be solved in terms of an antisymmetric tensor B_{mn} and a vector V_m

$$\begin{aligned}
H^m &= \varepsilon^{mnkl}\partial_nB_{kl} + \kappa\,\mathrm{tr}\left\{\varepsilon^{mnkl}(A_nF_{kl} - \frac{2}{3}A_nA_kA_l)\right. \\
&\qquad \left. - \frac{1}{2}\lambda^i\sigma^m\bar\lambda_i\right\} \\
G_{mn} &= \partial_mV_n - \partial_nV_m + 2\kappa\,\mathrm{tr}\left\{(X+\bar{X})F_{mn} + \frac{1}{4}\lambda^i\sigma_{mn}\lambda_i\right. \\
&\qquad \left. - \frac{1}{4}\bar\lambda_i\bar\sigma_{mn}\bar\lambda^i\right\}\,.
\end{aligned} \tag{46}$$

Because of the constraints (41) and (42), the superfield $\mathcal{D}^+_\alpha \mathbb{L}$ is no longer analytic. But also the Lagrangians (35) and (38) can be deformed to obtain supersymmetric actions with Chern-Simons interactions. Similarly to [9], let us introduce the following real superfield

$$\Sigma = \mathbb{L} - \frac{\kappa}{2}\,\mathrm{tr}\,\left(W - \bar{W}\right)^2 \ . \tag{47}$$

Using the Bianchi identities [13,14]

$$\bar{\mathcal{D}}^+_{\dot\alpha} W = 0 \qquad (\mathcal{D}^+)^2 W = (\bar{\mathcal{D}}^+)^2 \bar{W} \tag{48}$$

one can prove the important identities

$$\mathcal{D}^+_\alpha \bar{\mathcal{D}}^+_{\dot\beta} \Sigma = 0 \tag{49}$$

$$\begin{aligned}(\mathcal{D}^+)^2 \Sigma &= -(\bar{\mathcal{D}}^+)^2 \Sigma \\ &= \frac{\kappa}{2}\left\{(\bar{\mathcal{D}}^+)^2 \mathrm{tr}\,\left(\bar{W}^2\right) - (\mathcal{D}^+)^2 \mathrm{tr}\,\left(W^2\right)\right\}. \end{aligned}\tag{50}$$

Therefore, the imaginary superfield

$$\begin{aligned}\mathcal{L}^{++}_{\mathrm{vt}} = \frac{1}{4}\big\{&\mathcal{D}^{+\alpha}\Sigma \mathcal{D}^+_\alpha \Sigma + \Sigma (\mathcal{D}^+)^2 \Sigma \\ &-\bar{\mathcal{D}}^+_{\dot\alpha}\Sigma \bar{\mathcal{D}}^{+\dot\alpha}\Sigma\big\}\end{aligned}\tag{51}$$

satisfies both the constraints (6) and (7), and therefore can be used to construct a supersymmetric action. The corresponding action functional obtained by the rule (9) describes the Chern-Simons coupling of the vector-tensor multiplet to the $N = 2$ gauge multiplet. It was first derived in component approach [4] and then in $N = 2$ superspace [9]. We give only the bosonic part of the component Lagrangian:

$$\begin{aligned}\mathcal{L}_{\mathrm{vt}} = &\frac{1}{2}\partial^m \Phi\, \partial_m \Phi - \frac{1}{2} H^m H_m - \frac{1}{4} G^{mn} G_{mn} + \frac{1}{2} D^2 \\ &+ \mathrm{i}\kappa\, G^{mn}\, \mathrm{tr}\left\{(X - \bar{X})\tilde{F}_{mn}\right\} \\ &- \mathrm{i}\kappa\, H^m\, \mathrm{tr}\left\{(X - \bar{X})\mathcal{D}_m (X + \bar{X})\right\} \\ &- 2\kappa\left(\Phi - \frac{\kappa}{2}\mathrm{tr}\,(X - \bar{X})^2\right)\mathrm{tr}\left\{\mathcal{D}^m \bar{X}\, \mathcal{D}_m X\right. \\ &\left.- \frac{1}{2} F^{mn} F_{mn} - \frac{1}{2} Y^{ij} Y_{ij} + \frac{1}{4}[X, \bar{X}]^2\right\} \\ &+ 2\kappa^2\, \mathrm{tr}\left\{(X - \bar{X})\mathcal{D}^m \bar{X}\right\} \mathrm{tr}\left\{(X - \bar{X})\mathcal{D}_m X\right\} \\ &- \kappa^2\, \mathrm{tr}\left\{(X - \bar{X})F^{mn}\right\} \mathrm{tr}\left\{(X - \bar{X})F_{mn}\right\} \\ &- \kappa^2\, \mathrm{tr}\left\{(X - \bar{X})Y^{ij}\right\} \mathrm{tr}\left\{(X - \bar{X})Y_{ij}\right\} \\ &+ \text{fermionic terms}\ .\end{aligned}\tag{52}$$

Now, we generalize the total derivative Lagrangian (38) (which is an $N = 2$ analog of $\tilde{F}\,F$ or θ-term). Similar to [9], we introduce the real superfield

$$\Omega = \mathbb{L} + \frac{\kappa}{2}\,\mathrm{tr}\,\left(W + \bar{W}\right)^2 \ . \tag{53}$$

Its properties read

$$\mathcal{D}^+_\alpha \bar{\mathcal{D}}^+_{\dot\beta} \Omega = 0 \tag{54}$$

$$\begin{aligned}(\mathcal{D}^+)^2 \Omega &= (\bar{\mathcal{D}}^+)^2 \Omega \\ &= \frac{\kappa}{2}\left\{(\bar{\mathcal{D}}^+)^2 \mathrm{tr}\,\left(\bar{W}^2\right) + (\mathcal{D}^+)^2 \mathrm{tr}\,\left(W^2\right)\right\}. \end{aligned}\tag{55}$$

As a consequence, the imaginary superfield

$$\begin{aligned}\mathcal{L}^{++}_{\mathrm{der}} = \frac{\mathrm{i}}{4}\big\{&\mathcal{D}^{+\alpha}\Omega \mathcal{D}^+_\alpha \Omega + \Omega (\mathcal{D}^+)^2 \Omega \\ &+\bar{\mathcal{D}}^+_{\dot\alpha}\Omega \bar{\mathcal{D}}^{+\dot\alpha}\Omega\big\}\end{aligned}\tag{56}$$

respects both the constraints (6) and (7) and therefore defines a supersymmetric action.

The Lagrangian (56) is the deformation of (38). It is therefore a deformation of the Chern-Simons form $\tilde{F}\,F$ which carries topological information. In components, the bosonic Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\mathrm{der,bos}} = \partial_m \Big[&\left(\Phi + \frac{\kappa}{2}\mathrm{tr}\,(X + \bar{X})^2\right) \\ &\times \left(H^m - \mathrm{i}\kappa\, \mathrm{tr}\left\{(X + \bar{X})\mathcal{D}^m (X - \bar{X})\right\}\right) \\ &+ \frac{1}{2}\varepsilon^{mnkl} V_n \partial_k V_l \Big]\,.\end{aligned}\tag{57}$$

and contains total derivative terms only.

In summary, in the present paper we have described the vector-tensor multiplet and its Chern-Simons coupling to the $N = 2$ gauge multiplet in harmonic superspace. It would be of interest to find an unconstrained prepotential superfield formulation for the vector-tensor multiplet, which may exist, similar to the Fayet-Sohnius hypermultiplet, in harmonic superspace only.

After this work had appeared on the hep-th archive we became aware of a recent paper [16] where the authors presented a two-form formulation of the vector-tensor multiplet in central charge superspace and derived its coupling to the non-Abelian supergauge multiplet via the Chern-Simons form. The later paper is a natural development of the research started in [8].

Acknowledgements. This work was supported by the RFBR-DFG project No 96-02-00180, the RFBR project No 96-02-16017 and by the Alexander von Humboldt Foundation.

References

1. M.F. Sohnius, K.S. Stelle and P. West, Phys. Lett. B**92** (1980) 123; Nucl. Phys. B**173** (1980) 127
2. B. de Wit, V. Kaplunovsky, J. Louis and D. Lüst, Nucl. Phys. B**451** (1995) 53
3. P. Claus, B. de Wit, M. Faux, B. Kleijn, R. Siebelink and P. Termonia, Phys. Lett. B**373** (1996) 81
4. P. Claus, B. de Wit, M. Faux and P. Termonia, Nucl. Phys. B**491** (1997) 201
5. P. Fayet, Nucl. Phys. B**113** (1976) 135; A. Salam and J. Strathdee, Nucl. Phys. B**97** (1975) 293
6. M.F. Sohnius, Nucl. Phys. B**138** (1978) 109
7. A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky and E. Sokatchev, Class. Quantum Grav. **1** (1984) 469
8. A. Hindawi, B.A. Ovrut and D. Waldram, Phys. Lett. B**392** (1997) 85
9. R. Grimm, M. Hasler and C. Herrmann, The $N = 2$ vector-tensor multiplet, central charge superspace and Chern-Simons couplings, hep-th/9706108

10. P.S. Howe, K.S. Stelle and P.K. Townsend, Nucl. Phys. B**191** (1981) 445
11. G. Sierra and P.K. Townsend, in : Supersymmetry and Supergravity 1983, Proc. XIXth Winter School, Karpacz, ed. B. Milewski (World Scientific, 1983), p. 396;
B. de Wit, P.G. Lauwers, R. Philippe, S.-Q. Su and A. Van Proyen, Phys. Lett. B**134** (1984) 37;
S.J. Gates, Nucl. Phys. B**238** (1984) 349
12. N. Seiberg, Phys. Lett. B**206** (1988) 75
13. R. Grimm, M. Sohnius and J. Wess, Nucl. Phys. B**133** (1978) 275
14. M. Sohnius, Nucl. Phys. B**136** (1978) 461
15. E. Witten, Phys. Lett. B**86** (1979) 283
16. I. Buchbinder, A. Hindawi and B.A. Ovrut, A two-form formulation of the vector-tensor multiplet in central charge superspace, hep-th/9706216